Math 143 Sample Problems for Midterm Exam

Question 1 For each of the following sequences $\{a_n\}$, decide whether it converges or diverges and circle the appropriate word. If the sequence converges, compute the limit of the sequence and write the limit of the sequence in the blank. (Show all work.)

a) $a_n = \dfrac{\sqrt{2n^2+10n-1}}{4-6n}$ **Converges** **Diverges** **Limit=** _____

b) $a_n = \pi^{(3-\cos(3/n))}$ **Converges** **Diverges** **Limit=** _____

c) $a_n = \sin\left(\dfrac{\pi}{2} + (-1)^n \pi\right)$ **Converges** **Diverges** **Limit=** _____

d) $a_n = \dfrac{(n-1)!}{(n+1)!} + \left(\dfrac{n+6}{n-1}\right)^n$ **Converges** **Diverges** **Limit=** _____

Question 2 For each of the following series decide whether the series converges or diverges and circle the appropriate word. Write the name of the test used to decide in the blank. (Show all work.)

a) $\displaystyle\sum_{n=1}^{\infty} \left(2 - e^{(-1/n)}\right)$ **Converges** **Diverges** **Test Used=**_____

b) $\displaystyle\sum_{n=1}^{\infty} \frac{4\sqrt[3]{n^5}}{n^2}$ **Converges** **Diverges** **Test Used=**_____

c) $\displaystyle\sum_{n=1}^{\infty} \frac{2^{3n}}{10^n}$ **Converges** **Diverges** **Test Used=**_____

d) $\displaystyle\sum_{n=3}^{\infty} \frac{n^4}{(4n^5-9)^{6/5}}$ **Converges** **Diverges** **Test Used=**_____

e) $\displaystyle\sum_{n=0}^{\infty} \frac{5n-3}{n^2-2n+43}$ **Converges** **Diverges** **Test Used=**_____

f) $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2\, 3^n}{(2n+1)!}$ **Converges** **Diverges** **Test Used=**_____

Question 3 Compute the sum of the following infinite series:

a) $\displaystyle\sum_{n=3}^{\infty} \frac{(-3)^{n-2}}{7^{n+1}}$

b) $\displaystyle\sum_{n=2}^{\infty} \frac{4}{n(n+2)}$

Question 4 Does the following series converge conditionally, absolutely or diverge:

$$\sum_{n=1}^{\infty} (-1)^n \frac{(\ln n)^{2002} + \sin^2 n}{\sqrt[5]{n^{13}+11}}$$

Question 5 Find the interval of convergence of the power series (Don't forget to check the endpoints!):

$$\sum_{n=0}^{\infty} (-1)^n \frac{(3x+1)^n}{4^{2n}(n+1)}$$

Question 6 Approximate the definite integral $\displaystyle\int_0^1 x\cos(3x^3)\,dx$ by using the first three non-zero terms of the Maclaurin series for $f(x) = x\cos(3x^3)$. What is the maximum error in your approximation? You will need to know that $\cos x = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$.

Question 7 Find the Taylor series for $f(x) = \cos x$ centered at the point $x = \pi/4$.